Lesson 1: Modeling Linear Relationships

Classwork

Example 1: Logging On

Lenore has just purchased a tablet computer, and she is considering purchasing an Internet access plan so that she can connect to the Internet wirelessly from virtually anywhere in the world. One company offers an Internet access plan so that when a person connects to the company’s wireless network, the person is charged a fixed access fee for connecting plus an amount for the number of minutes connected based upon a constant usage rate in dollars per minute.

Lenore is considering this company’s plan, but the company’s advertisement does not state how much the fixed access fee for connecting is, nor does it state the usage rate. However, the company’s website says that a 10-minute session costs $0.40, a 20-minute session costs $0.70, and a 30-minute session costs $1.00. Lenore decides to use these pieces of information to determine both the fixed access fee for connecting and the usage rate.

Exercises 1–6

1. Lenore makes a table of this information and a graph where number of minutes is represented by the horizontal axis and total session cost is represented by the vertical axis. Plot the three given points on the graph. These three points appear to lie on a line. What information about the access plan suggests that the correct model is indeed a linear relationship?

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>Total Session Cost (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.40</td>
</tr>
<tr>
<td>20</td>
<td>0.70</td>
</tr>
<tr>
<td>30</td>
<td>1.00</td>
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<tr>
<td>40</td>
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<td>50</td>
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<tr>
<td>60</td>
<td></td>
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</tbody>
</table>
2. The rate of change describes how the total cost changes with respect to time.
   a. When the number of minutes increases by 10 (e.g., from 10 minutes to 20 minutes or from 20 minutes to 30 minutes), how much does the charge increase?

   b. Another way to say this would be the usage charge per 10 minutes of use. Use that information to determine the increase in cost based on only 1 minute of additional usage. In other words, find the usage charge per minute of use.

3. The company’s pricing plan states that the usage rate is constant for any number of minutes connected to the Internet. In other words, the increase in cost for 10 more minutes of use (the value that you calculated in Exercise 2) is the same whether you increase from 20 to 30 minutes, 30 to 40 minutes, etc. Using this information, determine the total cost for 40 minutes, 50 minutes, and 60 minutes of use. Record those values in the table, and plot the corresponding points on the graph in Exercise 1.

4. Using the table and the graph in Exercise 1, compute the hypothetical cost for 0 minutes of use. What does that value represent in the context of the values that Lenore is trying to figure out?

5. On the graph in Exercise 1, draw a line through the points representing 0 to 60 minutes of use under this company’s plan. The slope of this line is equal to the rate of change, which in this case is the usage rate.

6. Using $x$ for the number of minutes and $y$ for the total cost in dollars, write a function to model the linear relationship between minutes of use and total cost.
Example 2: Another Rate Plan

A second wireless access company has a similar method for computing its costs. Unlike the first company that Lenore was considering, this second company explicitly states its access fee is $0.15, and its usage rate is $0.04 per minute.

Total Session Cost = $0.15 + $0.04 (number of minutes)

Exercises 7–16

7. Let \( x \) represent the number of minutes used and \( y \) represent the total session cost in dollars. Construct a linear function that models the total session cost based on the number of minutes used.

8. Using the linear function constructed in Exercise 7, determine the total session cost for sessions of \( 0, 10, 20, 30, 40, 50, \) and \( 60 \) minutes, and fill in these values in the table below.

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>Total Session Cost (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
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<td>50</td>
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<td>60</td>
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</tbody>
</table>

9. Plot these points on the original graph in Exercise 1, and draw a line through these points. In what ways does the line that represents this second company’s access plan differ from the line that represents the first company’s access plan?
MP3 download sites are a popular forum for selling music. Different sites offer pricing that depends on whether or not you want to purchase an entire album or individual songs à la carte. One site offers MP3 downloads of individual songs with the following price structure: a $3 fixed fee for a monthly subscription plus a charge of $0.25 per song.

10. Using $x$ for the number of songs downloaded and $y$ for the total monthly cost in dollars, construct a linear function to model the relationship between the number of songs downloaded and the total monthly cost.

11. Using the linear function you wrote in Exercise 10, construct a table to record the total monthly cost (in dollars) for MP3 downloads of 10 songs, 20 songs, and so on up to 100 songs.

<table>
<thead>
<tr>
<th>Number of Songs</th>
<th>Total Monthly Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$3.25</td>
</tr>
<tr>
<td>20</td>
<td>$6.50</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>100</td>
<td>$325.00</td>
</tr>
</tbody>
</table>

12. Plot the 10 data points in the table on a coordinate plane. Let the $x$-axis represent the number of songs downloaded and the $y$-axis represent the total monthly cost (in dollars) for MP3 downloads.
A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid $200. If 70 tickets are sold, the band will be paid $260.

13. Determine the rate of change.

14. Let \( x \) represent the number of tickets sold and \( y \) represent the amount the band will be paid in dollars. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

15. What flat fee will the band be paid for playing the concert regardless of the number of tickets sold?

16. How much will the band receive for each ticket sold?
Lesson 1: Modeling Linear Relationships

Problem Set

1. Recall that Lenore was investigating two wireless access plans. Her friend in Europe says that he uses a plan in which he pays a monthly fee of 30 euro plus 0.02 euro per minute of use.
   a. Construct a table of values for his plan's monthly cost based on 100 minutes of use for the month, 200 minutes of use, and so on up to 1,000 minutes of use. (The charge of 0.02 euro per minute of use is equivalent to 2 euro per 100 minutes of use.)
   b. Plot these 10 points on a carefully labeled graph, and draw the line that contains these points.
   c. Let \( x \) represent minutes of use and \( y \) represent the total monthly cost in euro. Construct a linear function that determines monthly cost based on minutes of use.
   d. Use the function to calculate the cost under this plan for 750 minutes of use. If this point were added to the graph, would it be above the line, below the line, or on the line?

2. A shipping company charges a $4.45 handling fee in addition to $0.27 per pound to ship a package.
   a. Using \( x \) for the weight in pounds and \( y \) for the cost of shipping in dollars, write a linear function that determines the cost of shipping based on weight.
   b. Which line (solid, dotted, or dashed) on the following graph represents the shipping company's pricing method? Explain.

Lesson Summary

A linear function can be used to model a linear relationship between two types of quantities. The graph of a linear function is a straight line.

A linear function can be constructed using a rate of change and an initial value. It can be interpreted as an equation of a line in which:
- The rate of change is the slope of the line and describes how one quantity changes with respect to another quantity.
- The initial value is the \( y \)-intercept.
3. Kelly wants to add new music to her MP3 player. Another subscription site offers its downloading service using the following: Total Monthly Cost = 5.25 + 0.30 (number of songs).
   a. Write a sentence (all words, no math symbols) that the company could use on its website to explain how it determines the price for MP3 downloads for the month.
   b. Let \( x \) represent the number of songs downloaded and \( y \) represent the total monthly cost in dollars. Construct a function to model the relationship between the number of songs downloaded and the total monthly cost.
   c. Determine the cost of downloading 10 songs.

4. Li Na is saving money. Her parents gave her an amount to start, and since then she has been putting aside a fixed amount each week. After six weeks, Li Na has a total of $82 of her own savings in addition to the amount her parents gave her. Fourteen weeks from the start of the process, Li Na has $118.
   a. Using \( x \) for the number of weeks and \( y \) for the amount in savings (in dollars), construct a linear function that describes the relationship between the number of weeks and the amount in savings.
   b. How much did Li Na’s parents give her to start?
   c. How much does Li Na set aside each week?
   d. Draw the graph of the linear function below (start by plotting the points for \( x = 0 \) and \( x = 20 \)).
Lesson 2: Interpreting Rate of Change and Initial Value

Classwork

Linear functions are defined by the equation of a line. The graphs and the equations of the lines are important for understanding the relationship between the two variables represented in the following example as $x$ and $y$.

Example 1: Rate of Change and Initial Value

The equation of a line can be interpreted as defining a linear function. The graphs and the equations of lines are important in understanding the relationship between two types of quantities (represented in the following examples by $x$ and $y$).

In a previous lesson, you encountered an MP3 download site that offers downloads of individual songs with the following price structure: a $3 fixed fee for a monthly subscription plus a fee of $0.25 per song. The linear function that models the relationship between the number of songs downloaded and the total monthly cost of downloading songs can be written as

$$y = 0.25x + 3,$$

where $x$ represents the number of songs downloaded and $y$ represents the total monthly cost (in dollars) for MP3 downloads.

a. In your own words, explain the meaning of 0.25 within the context of the problem.

b. In your own words, explain the meaning of 3 within the context of the problem.

The values represented in the function can be interpreted in the following way:

$$y = 0.25x + 3$$

rate of change
initial value
The coefficient of $x$ is referred to as the \textit{rate of change}. It can be interpreted as the change in the values of $y$ for every one-unit increase in the values of $x$.

When the rate of change is positive, the linear function is \textit{increasing}. In other words, \textit{increasing} indicates that as the $x$-value increases, so does the $y$-value.

When the rate of change is negative, the linear function is \textit{decreasing}. \textit{Decreasing} indicates that as the $x$-value increases, the $y$-value decreases.

The constant value is referred to as the \textit{initial value} or $y$-intercept and can be interpreted as the value of $y$ when $x = 0$.

\textbf{Exercises 1–6: Is It a Better Deal?}

Another site offers MP3 downloads with a different price structure: a $2$ fixed fee for a monthly subscription plus a fee of $0.40$ per song.

1. Write a linear function to model the relationship between the number of songs downloaded and the total monthly cost. As before, let $x$ represent the number of songs downloaded and $y$ represent the total monthly cost (in dollars) of downloading songs.

2. Determine the cost of downloading $0$ songs and $10$ songs from this site.

3. The graph below already shows the linear model for the first subscription site (Company 1): $y = 0.25x + 3$. Graph the equation of the line for the second subscription site (Company 2) by marking the two points from your work in Exercise 2 (for $0$ songs and $10$ songs) and drawing a line through those two points.
4. Which line has a steeper slope? Which company’s model has the more expensive cost per song?

5. Which function has the greater initial value?

6. Which subscription site would you choose if you only wanted to download 5 songs per month? Which company would you choose if you wanted to download 10 songs? Explain your reasoning.

Exercises 7–9: Aging Autos

7. When someone purchases a new car and begins to drive it, the mileage (meaning the number of miles the car has traveled) immediately increases. Let \( x \) represent the number of years since the car was purchased and \( y \) represent the total miles traveled. The linear function that models the relationship between the number of years since purchase and the total miles traveled is \( y = 15000x \).
   a. Identify and interpret the rate of change.
   
   b. Identify and interpret the initial value.
c. Is the mileage increasing or decreasing each year according to the model? Explain your reasoning.

8. When someone purchases a new car and begins to drive it, generally speaking, the resale value of the car (in dollars) goes down each year. Let $x$ represent the number of years since purchase and $y$ represent the resale value of the car (in dollars). The linear function that models the resale value based on the number of years since purchase is $y = 20000 - 1200x$.
   a. Identify and interpret the rate of change.

   b. Identify and interpret the initial value.

   c. Is the resale value increasing or decreasing each year according to the model? Explain.

9. Suppose you are given the linear function $y = 2.5x + 10$.
   a. Write a story that can be modeled by the given linear function.

   b. What is the rate of change? Explain its meaning with respect to your story.

   c. What is the initial value? Explain its meaning with respect to your story.
Lesson Summary

When a linear function is given by the equation of a line of the form \( y = mx + b \), the rate of change is \( m \), and the initial value is \( b \). Both are easy to identify.

The rate of change of a linear function is the slope of the line it represents. It is the change in the values of \( y \) per a one-unit increase in the values of \( x \).

- A positive rate of change indicates that a linear function is increasing.
- A negative rate of change indicates that a linear function is decreasing.
- Given two lines each with positive slope, the function represented by the steeper line has a greater rate of change.

The initial value of a linear function is the value of the \( y \)-variable when the \( x \)-value is zero.

Problem Set

1. A rental car company offers the following two pricing methods for its customers to choose from for a one-month rental:
   - Method 1: Pay $400 for the month, or
   - Method 2: Pay $0.30 per mile plus a standard maintenance fee of $35.
   a. Construct a linear function that models the relationship between the miles driven and the total rental cost for Method 2. Let \( x \) represent the number of miles driven and \( y \) represent the rental cost (in dollars).
   b. If you plan to drive 1,100 miles for the month, which method would you choose? Explain your reasoning.

2. Recall from a previous lesson that Kelly wants to add new music to her MP3 player. She was interested in a monthly subscription site that offered its MP3 downloading service for a monthly subscription fee plus a fee per song. The linear function that modeled the total monthly cost in dollars (\( y \)) based on the number of songs downloaded (\( x \)) is \( y = 5.25 + 0.30x \).
   The site has suddenly changed its monthly price structure. The linear function that models the new total monthly cost in dollars (\( y \)) based on the number of songs downloaded (\( x \)) is \( y = 0.35x + 4.50 \).
   a. Explain the meaning of the value 4.50 in the new equation. Is this a better situation for Kelly than before?
   b. Explain the meaning of the value 0.35 in the new equation. Is this a better situation for Kelly than before?
   c. If you were to graph the two equations (old versus new), which line would have the steeper slope? What does this mean in the context of the problem?
   d. Which subscription plan provides the better value if Kelly downloads fewer than 15 songs per month?
Lesson 3: Representations of a Line

Classwork

Example 1: Rate of Change and Initial Value Given in the Context of the Problem

A truck rental company charges a $150 rental fee in addition to a charge of $0.50 per mile driven. Graph the linear function relating the total cost of the rental in dollars, $C$, to the number of miles driven, $m$, on the axes below.

a. If the truck is driven 0 miles, what is the cost to the customer? How is this shown on the graph?

b. What is the rate of change that relates cost to number of miles driven? Explain what it means within the context of the problem.

c. On the axes given, sketch the graph of the linear function that relates $C$ to $m$.

d. Write the equation of the linear function that models the relationship between number of miles driven and total rental cost.
Exercises

Jenna bought a used car for $18,000. She has been told that the value of the car is likely to decrease by $2,500 for each year that she owns the car. Let the value of the car in dollars be $V$ and the number of years Jenna has owned the car be $t$.

1. What is the value of the car when $t = 0$? Show this point on the graph.

2. What is the rate of change that relates $V$ to $t$? (Hint: Is it positive or negative? How can you tell?)

3. Find the value of the car when:
   a. $t = 1$
   b. $t = 2$
   c. $t = 7$

4. Plot the points for the values you found in Exercise 3, and draw the line (using a straightedge) that passes through those points.

5. Write the linear function that models the relationship between the number of years Jenna has owned the car and the value of the car.
An online bookseller has a new book in print. The company estimates that if the book is priced at $15, then 800 copies of the book will be sold per day, and if the book is priced at $20, then 550 copies of the book will be sold per day.

6. Identify the ordered pairs given in the problem. Then, plot both on the graph.

7. Assume that the relationship between the number of books sold and the price is linear. (In other words, assume that the graph is a straight line.) Using a straightedge, draw the line that passes through the two points.

8. What is the rate of change relating number of copies sold to price?

9. Based on the graph, if the company prices the book at $18, about how many copies of the book can they expect to sell per day?

10. Based on the graph, approximately what price should the company charge in order to sell 700 copies of the book per day?
Lesson Summary

When the rate of change, $b$, and an initial value, $a$, are given in the context of a problem, the linear function that models the situation is given by the equation $y = a + bx$.

The rate of change and initial value can also be used to sketch the graph of the linear function that models the situation.

When two or more ordered pairs are given in the context of a problem that involves a linear relationship, the graph of the linear function is the line that passes through those points. The linear function can be represented by the equation of that line.

Problem Set

1. A plumbing company charges a service fee of $120, plus $40 for each hour worked. Sketch the graph of the linear function relating the cost to the customer (in dollars), $C$, to the time worked by the plumber (in hours), $t$, on the axes below.

   ![Graph](image)

   a. If the plumber works for 0 hours, what is the cost to the customer? How is this shown on the graph?

   b. What is the rate of change that relates cost to time?

   c. Write a linear function that models the relationship between the hours worked and the cost to the customer.

   d. Find the cost to the customer if the plumber works for each of the following number of hours.

      i. 1 hour
      ii. 2 hours
      iii. 6 hours

   e. Plot the points for these times on the coordinate plane, and use a straightedge to draw the line through the points.
2. An author has been paid a writer’s fee of $1,000 plus $1.50 for every copy of the book that is sold.
   a. Sketch the graph of the linear function that relates the total amount of money earned in dollars, \( A \), to the number of books sold, \( n \), on the axes below.

   \[
   \begin{array}{c|c}
   \text{Number of Books Sold} & \text{Total Amount of Money Earned in Dollars} \\
   \hline
   0 & 1000 \\
   200 & 1200 \\
   400 & 1400 \\
   600 & 1600 \\
   800 & 1800 \\
   1000 & 2000 \\
   \hline
   \end{array}
   \]

   b. What is the rate of change that relates the total amount of money earned to the number of books sold?
   c. What is the initial value of the linear function based on the graph?
   d. Let the number of books sold be \( n \) and the total amount earned be \( A \). Construct a linear function that models the relationship between the number of books sold and the total amount earned.