Lesson Objectives
Find the perimeter and area of rectangles and parallelograms

Vocabulary
perimeter (p. 388)
area (p. 389)

Additional Examples

Example 1
Find the perimeter of each figure.

A. 

\[ P = \square + \square + \square + \square \] 
Add all side lengths.

\[ = \square \text{ units} \]

or \[ P = 2b + 2h \] 
Perimeter of rectangle

\[ = 2(\square) + 2(\square) \] 
Substitute \square for \( b \) and \square for \( h \).

\[ = 28 + 10 = \square \text{ units} \]

B. 

\[ P = \square + \square + \square + \square \]

\[ = \square \text{ units} \]
Example 2

Graph and find the area of the figure with the given vertices.

A. \((-1, -2), (2, -2), (2, 3), (-1, 3)\)

\[ \text{A} = bh \quad \text{Area of a rectangle} \]

\[ = \square \cdot \square \quad \text{Substitute } \square \text{ for } b \text{ and } \square \text{ for } h. \]

\[ = \square \text{ units}^2 \]

Example 3

Find the perimeter and area of the figure.

The length of the side that is not labeled is the same as the total length of the opposite side, or \(\square + \square + \square\), which equals \(\square\).

\[ P = \square + \square + \square + \square + \square + \square + \square + \square \]

\[ = \square \text{ units} \]

The length of the middle rectangle (above) is \(5 - \square = \square\).

\[ A = (6 \cdot \square) + (6 \cdot \square) + (6 \cdot \square) \quad \text{Add the areas together.} \]

\[ = \square + \square + \square \]

\[ = \square \text{ square units} \]
Lesson Objectives
Make scale models of solid figures

Vocabulary
capacity (p. 440)

Additional Examples

Example 1
A 3 cm cube is built from small cubes, each 1 cm on an edge. Compare the following values.

A. the edge lengths of the large and small cubes

\[
\frac{3 \text{ cm cube}}{1 \text{ cm cube}} \rightarrow \frac{3 \text{ cm}}{1 \text{ cm}} = \text{ cm}
\]

Ratio of corresponding cm

The edges of the large cube are times as long as the edges of the small cube.

Example 2
A box is in the shape of a rectangular prism. The box is 4 ft tall, and its base has a length of 3 ft and a width of 2 ft. For a 6 in. tall model of the box, find the following.

A. What is the scale factor of the model?

\[
\frac{6 \text{ in.}}{4 \text{ ft}} = \frac{6 \text{ in.}}{4 \text{ ft}} = \text{ in.}
\]

Convert and simplify.

The scale factor of the model is .
Example 3

It takes 30 seconds for a pump to fill a cubic container whose edge measures 1 ft. How long does it take for the pump to fill a cubic container whose edge measures 2 ft?

\[ V = \square \text{ ft} \cdot \square \text{ ft} \cdot \square \text{ ft} = \square \text{ ft}^3 \]

Find the volume of the larger container.

Set up a proportion and solve.

\[ \frac{30}{1 \text{ ft}^3} = \frac{x}{\square \text{ ft}^3} \]

Cancel units.

Multiply.

\[ \square \cdot \square = \square \]

Calculate the fill time.

It takes \square seconds, or \square minutes to fill the larger container.

Try This

1. A 2 cm cube is built from small cubes, each 1 cm on an edge. Compare the following values.
   - the edge lengths of the large and small cubes

2. A box is in the shape of a rectangular prism. The box is 8 ft tall, and its base has a length of 6 ft and a width of 4 ft. For a 6 in. tall model of the box, find the following.
   - What is the scale factor of the model?

3. It takes 45 seconds for a machine to fill a cubic box whose edge measures 1 yd. How long does it take for the machine to fill a cubic box whose edge measures 5 yd?
Lesson Objectives

Find the perimeter and area of triangles and trapezoids

Additional Examples

Example 1

Find the perimeter of each figure.

A. \[ P = \underline{4} + \underline{7} + \underline{10} \]

Add all sides.

\[ P = \underline{21} \text{ units} \]

B. \[ P = \underline{8} + \underline{11} + \underline{17} + \underline{6} \]

Add all sides.

\[ P = \underline{42} \text{ units} \]

Example 2

Find the missing measurement for the trapezoid with perimeter 71 in.

\[ P = \underline{15} + \underline{18} + \underline{22} + d \]

\[ 71 = \underline{55} + d \]

Substitute \( P \) for \( P \).

\[ 16 = d \]

Subtract \( \underline{55} \) from both sides.

\[ d = \underline{16} \]
Lesson Objectives
Find the circumference and area of circles

Vocabulary

- circle (p. 400)

- radius (p. 400)

- diameter (p. 400)

- circumference (p. 400)

Additional Examples

Example 1
Find the circumference of each circle, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).

A. circle with a radius of 4 m

\[ C = 2\pi r \]
\[ = 2\pi(4) \]
\[ = \pi \approx 12.56 \text{ m} \]

B. circle with a diameter of 3.3 ft

\[ C = \pi d \]
\[ = \pi(3.3) \]
\[ = \pi \approx 10.47 \text{ ft} \]

Example 2
Find the area of each circle, both in terms of \( \pi \) and to the nearest tenth. Use 3.14 for \( \pi \).

A. circle with a radius of 4 in.

\[ A = \pi r^2 \]
\[ = \pi(4^2) \]
\[ = \pi \approx 50.27 \text{ in}^2 \]

B. circle with a diameter of 3.3 m

\[ A = \pi r^2 \]
\[ = \pi(\frac{3.3}{2})^2 \]
\[ = \pi \approx 17.67 \text{ m}^2 \]
Example 3

Graph the circle with center \((-2, 1)\) that passes through \((1, 1)\). Find the area and circumference, both to the nearest tenth. Use 3.14 for \(\pi\).

\[ A = \pi r^2 \]
\[ = \pi (\phantom{0})^2 \]
\[ = \phantom{0}\pi \text{ units}^2 \]
\[ \approx \phantom{0}\pi \text{ units}^2 \]

\[ C = \pi d \]
\[ = \pi (\phantom{0}) \]
\[ = \phantom{0}\pi \text{ units} \]
\[ \approx \phantom{0}\pi \text{ units} \]

Example 4

A Ferris wheel has a diameter of 56 feet and makes 15 revolutions per ride. How far would someone travel during a ride? Use \(\frac{22}{7}\) for \(\pi\).

\[ C = \pi d = \pi (\phantom{0}) \approx \frac{22}{7} (\phantom{0}) = \phantom{0} \]

Find the circumference.

The distance traveled is the circumference of the Ferris wheel time the number of revolutions, or about \(\phantom{0} \cdot \phantom{0} \approx \phantom{0}\) ft.

Try This

1. Find the circumference of the circle, both in terms of \(\pi\) and to the nearest tenth. Use 3.14 for \(\pi\).

   circle with a diameter of 4.25 in.
Lesson Objectives
Draw and identify parts of three-dimensional figures

Vocabulary
face (p. 408)
edge (p. 408)
vertex (p. 408)
orthogonal views (p. 408)

Additional Examples

Example 1
Name the vertices, edges, and faces of the three-dimensional figure shown.

The vertices are: ________________.

The edges are: ________________.

The faces are triangles ________________, and rectangles ________________.
Example 2

Draw the figure shown in the front, top, and side views.

From the front and sides views, there appears to be cube in the top level, in the corner. The top view shows the bottom layer has cubes.

Example 3

Draw the front, top, and side views of the figure.

Front: The figure looks like a row of 3 squares on the bottom with square on top of the leftmost square and 2 squares on top of the rightmost square.

Top: The figure looks like a row of squares.

Side: The figure looks like a column of squares.
Lesson Objectives
Find the volume of prisms and cylinders

Vocabulary
prism (p. 413)
cylinder (p. 413)

Additional Examples

Example 1
Find the volume of each figure to the nearest tenth. Use 3.14 for π.

A. A rectangular prism with base 2 cm by 5 cm and height 3 cm.

\[ B = \boxed{} \times \boxed{} = \boxed{} \text{ cm}^2 \]

Area of \( B \) of a prism

\[ V = Bh \]

\[ = \boxed{} \times \boxed{} \]

\[ = \boxed{} \text{ cm}^3 \]

B. A cylinder with radius 2 in. and height 4 in.

\[ B = \pi(\boxed{}^2) = \boxed{} \pi \text{ in}^2 \]

Volume of a \( \boxed{} \) of base

\[ V = Bh \]

\[ = \boxed{} \pi \times \boxed{} \]

\[ = \boxed{} \pi \approx \boxed{} \text{ in}^3 \]
Example 2

A. A juice box measures 3 in. by 2 in. by 4 in. Explain whether tripling the length, width, or height of the box would triple the amount of juice the box holds.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Double the Length</th>
<th>Double the Width</th>
<th>Double the Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = lwh )</td>
<td>( V = (2l)wh )</td>
<td>( V = l(2w)h )</td>
<td>( V = lw(2h) )</td>
</tr>
<tr>
<td>( = )</td>
<td>( = )</td>
<td>( = )</td>
<td>( = )</td>
</tr>
<tr>
<td>( \text{in}^3 )</td>
<td>( \text{in}^3 )</td>
<td>( \text{in}^3 )</td>
<td>( \text{in}^3 )</td>
</tr>
</tbody>
</table>

The original box has a volume of \( \text{in}^3 \). You could triple the volume to \( \text{in}^3 \) by tripling any one of the dimensions. So tripling the length, width, or height \( \text{in}^3 \) triple the amount of juice the box holds.

Example 3

A drum company advertises a snare drum that is 4 inches high and 12 inches in diameter. Estimate the volume of the drum.

\[
r = \frac{d}{2} = \frac{12}{2} = \text{in}
\]

\[
V = (\pi r^2)h
\]

\[
= (3.14)(\text{in}^2)(4)
\]

\[
= \text{in} \cdot 4
\]

\[
\approx \text{in}^3
\]
Example 4

Find the volume of the barn.

\[
\text{Volume of barn} = \text{Volume of rectangular prism} + \text{Volume of triangular prism}
\]

\[
V = (40)(50)(15) + \frac{1}{2}(40)(10)(50)
\]

\[
= \underline{ } + \underline{ }
\]

\[
= \underline{ } \text{ ft}^3
\]

The volume is 40,000 ft\(^3\).

Try This

1. Find the volume of the figure to the nearest tenth.

2. A garbage container measures 2 ft by 3 ft by 7 ft. Explain whether doubling the length, width, or height of the container would double the amount of garbage the container holds.

3. A drum company advertises a bass drum that is 9 inches high and 19 inches in diameter. Estimate the volume of the drum.

4. Find the volume of the figure.
Lesson Objectives
Find the volume of pyramids and cones

Vocabulary
pyramid (p. 420)
cone (p. 420)

Additional Examples

Example 1
Find the volume of each figure. Use 3.14 for $\pi$.

A. $B = \frac{1}{2} \cdot \text{area} \quad \text{cm}^2$
$V = \frac{1}{3} \cdot \text{area} \cdot \text{height} \quad V = \frac{1}{3}Bh$
$V = \text{Volume} \quad \text{cm}^3$

B. $B = \pi \cdot \text{radius}^2 \quad \pi \text{in}^2$
$V = \frac{1}{3} \cdot \pi \cdot \text{radius} \cdot \text{height} \quad V = \frac{1}{3}Bh$
$V = \text{Volume} \quad \pi \approx \text{Volume} \quad \text{in}^3$ Use 3.14 for $\pi$.  


Example 2

A cone has a radius of 3 ft. and a height of 4 ft. Explain whether tripling the height would have the same effect on the volume of the cone as tripling the radius.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Triple the Height</th>
<th>Triple the Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ V = \frac{1}{3} \pi r^2 h ]</td>
<td>[ V = \frac{1}{3} \pi r^2 (3h) ]</td>
<td>[ V = \frac{1}{3} \pi (3r)^2 h ]</td>
</tr>
<tr>
<td>[ = \frac{1}{3} \pi (3^2) 4 ]</td>
<td>[ = \frac{1}{3} \pi (3^2) (3 \cdot 4) ]</td>
<td>[ = \frac{1}{3} \pi (3 \cdot 3)^2 (4) ]</td>
</tr>
<tr>
<td>[ \approx ]</td>
<td>[ \approx ]</td>
<td>[ \approx ]</td>
</tr>
</tbody>
</table>

When the height of the cone is tripled, the volume is \[ \text{ [ ] } \]. When the radius is tripled, the volume becomes \[ \text{ [ ] } \] times the original volume.

Example 3

The Pyramid of Kukulcán in Mexico is a square pyramid. Its height is 24 m and its base has 55 m sides. Find the volume of the pyramid.

\[ B = \left( \frac{55}{2} \right)^2 = \text{ [ ] } \text{ m}^2 \]

\[ A = bh \]

\[ V = \frac{1}{3} \left( \frac{55}{2} \right)^2 \left( \frac{24}{2} \right) = \text{ [ ] } \text{ m}^3 \]

\[ V = \frac{1}{3} bh \]

Example 4

Use a calculator to find the volume of a cone to the nearest cubic centimeter if the radius of the base is 15 cm and the height is 64 cm.

Use the \( \pi \) button on your calculator to find the area of the base.

\[ B = \pi r^2 \]

Next, with the area of the base still displayed, find the volume of the cone.

\[ V = \frac{1}{3} bh \]

The volume of the cone is approximately \[ \text{ [ ] } \text{ cm}^3 \].
Lesson Objectives
Find the surface area of prisms and cylinders

Vocabulary
- surface area (p. 427)
- lateral face (p. 427)
- lateral surface (p. 427)

Additional Examples

Example 1
Find the surface area of each figure.

A.

\[ S = 2\pi r^2 + 2\pi rh \]

\[ = 2\pi (\underline{4^2}) + 2\pi (\underline{4})(\underline{6}) \]

\[ = \underline{50.27} \text{ in}^2 \approx \underline{50} \text{ in}^2 \]

B.

\[ S = 2B + P = 2(\underline{10 \cdot 5 \cdot 3}) + (\underline{5})(\underline{6}) \]

\[ = \underline{132} \text{ ft}^2 \]
Example 2

A cylinder has diameter 8 in. and height 3 in. Explain whether tripling the height would have the same effect on the surface area as tripling the radius.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Triple the Height</th>
<th>Triple the Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 2\pi r^2 + 2\pi rh$</td>
<td>$S = 2\pi r^2 + 2\pi r(3h)$</td>
<td>$S = 2\pi(3r)^2 + 2\pi(3r)h$</td>
</tr>
<tr>
<td>$= 2\pi(4)^2 + 2\pi(4)(3)$</td>
<td>$= 2\pi(4)^2 + 2\pi(4)(9)$</td>
<td>$= 2\pi(12)^2 + 2\pi(12)(3)$</td>
</tr>
<tr>
<td>$= 56\pi \text{ in}^2 \approx \boxed{174}$</td>
<td>$= 104\pi \text{ in}^2 \approx \boxed{328}$</td>
<td>$= 360\pi \text{ in}^2 \approx \boxed{1131}$</td>
</tr>
</tbody>
</table>

They have the same effect. Tripling the radius would increase the surface area than tripling the height.

Example 3

A cylindrical soup can is 7.6 cm in diameter and 11.2 cm tall. What is the area of the label that covers the side of the can?

$L = 2\pi rh$

Only the lateral surface needs to be covered.

$diameter = 7.6$, so $r =$

$
= \boxed{2\pi(7.6})(\boxed{11.2})
= \boxed{79.77\pi} \approx \boxed{247.3} \text{ cm}^3$

Try This

1. Find the surface area of the figure.
Lesson Objectives
Find the surface area of pyramids and cones

Vocabulary
slant height (p. 432)

regular pyramid (p. 432)

right cone (p. 432)

Additional Examples

Example 1
Find the surface area of the figure to the nearest tenth.

A. 

\[ S = B + \frac{1}{2}Pl \]

\[ = (\text{ } \cdot \text{ } ) + \frac{1}{2}(\text{ })(\text{ }) \]

\[ = \text{ } \text{ft}^2 \]
Example 2

A cone has diameter 8 in. and slant height 3 in. Explain whether tripling the slant height would have the same effect on the surface area as tripling the radius.

<table>
<thead>
<tr>
<th>Original Dimensions</th>
<th>Triple the Slant Height</th>
<th>Triple the Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ S = \pi r^2 + \pi rl ]</td>
<td>[ S = \pi r^2 + \pi r(3l) ]</td>
<td>[ S = \pi(3r)^2 + \pi(3r)l ]</td>
</tr>
<tr>
<td>[ = \pi(4)^2 + \pi(4)(3) ]</td>
<td>[ = \pi(4)^2 + \pi(4)(9) ]</td>
<td>[ = \pi(12)^2 + \pi(12)(3) ]</td>
</tr>
<tr>
<td>[ = 28\pi \text{ in}^2 \approx ]</td>
<td>[ = 52\pi \text{ in}^2 \approx ]</td>
<td>[ = 180\pi \text{ in}^2 \approx ]</td>
</tr>
</tbody>
</table>

They have the same effect. Tripling the radius would increase the surface area than tripling the slant height.

Example 3

The upper portion of an hourglass is approximately an inverted cone with the given dimensions. What is the lateral surface area of the upper portion of the hourglass?

The slant height, radius, and depth of the hourglass form a triangle.

\[ a^2 + b^2 = l^2 \]

Pythagorean Theorem

\[ 24^2 + 10^2 = l^2 \]

\[ l = \sqrt{24^2 + 10^2} \]

\[ L = \pi rl \]

Lateral surface area

\[ = \pi(24)(l) = \pi \approx \text{mm}^2 \]
Lesson Objectives
Find the volume and surface area of spheres

Vocabulary
sphere (p. 436)

hemisphere (p. 436)

great circle (p. 436)

Additional Examples

Example 1
Find the volume of a sphere with radius 12 cm, both in terms of \( \pi \) and to the nearest tenth of a unit. Use 3.14 for \( \pi \).

\[ V = \frac{4}{3} \pi r^3 \] of a sphere

\[ = \frac{4}{3} \pi (\underline{12}^3) \] Substitute \( \underline{12} \) for \( r \).

\[ = \underline{602.88} \pi \text{ cm}^3 \approx \underline{1885} \text{ cm}^3 \]

Example 2
Find the surface area, both in terms of \( \pi \) and to the nearest tenth of a unit. Use 3.14 for \( \pi \).

\[ S = 4\pi r^2 \] area of a sphere

\[ = 4\pi (\underline{3}^2) \] Substitute \( \underline{3} \) for \( r \).

\[ = \underline{36} \pi \text{ in}^2 \approx \underline{113.1} \text{ in}^2 \]
Example 3

Compare the volumes and surface areas of a sphere with radius 42 cm and a rectangular prism measuring $\frac{44}{11547} \times \frac{84}{11547} \times \frac{84}{11547}$ cm.

**Sphere:**

$$V = \left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4}{3}\right)\pi \left(\frac{22}{7}\right)^3$$

$$\approx \frac{4}{3}\left(\frac{22}{7}\right)^3$$

$$\approx \frac{4}{3} \times 13829$$

$$\approx\boxed{18095.5}$$

$$S = 4\pi r^2 = 4\pi \left(\frac{22}{7}\right)^2$$

$$\approx \frac{4}{3} \times \frac{22}{7} \times \frac{22}{7}$$

$$\approx \boxed{376.9}$$

**Rectangular Prism:**

$$V = lwh$$

$$= \left(\frac{44}{11547}\right) \left(\frac{84}{11547}\right) \left(\frac{84}{11547}\right)$$

$$= \boxed{\frac{290304}{11547}}$$

$$S = 2lw + 2lh + 2wh$$

$$= 2\left(\frac{44}{11547}\right)\left(\frac{84}{11547}\right) + 2\left(\frac{44}{11547}\right)\left(\frac{84}{11547}\right) + 2\left(\frac{84}{11547}\right)\left(\frac{84}{11547}\right)$$

$$= \boxed{426.9}$$

The sphere and the prism have approximately the same , but the prism has a surface area.

Try This

1. Find the volume of a sphere with radius 3 cm, both in terms of $\pi$ and to the nearest tenth of a unit.

2. The moon has a radius of 1,738 km. Find the surface area, both in terms of $\pi$ and to the nearest tenth.